The problem of singularities caused by higher order curvature corrections in four dimensional string gravity

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Abstract. The influence of higher order (stringly inspired) curvature corrections to the classical General Relativity spherically symmetric solution is studied. In string gravity these curvature corrections have a special form and can provide a singular contribution to the field equations because they generate higher derivatives of metric functions multiplied by a small parameter. Analytically and numerically it is shown that sometimes in 4D string gravity the Schwarzschild solution is not recovered when the string coupling constant vanishes and limited number of higher order curvature corrections is considered.

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1. Introduction

The idea of higher order corrections to the Lagrangian of considered system became a rather common approach in modern theoretical physics. Sometimes, these additional corrections with higher order derivatives can drastically modify (through a singular contribution) the solutions of the corresponding Euler-Lagrange equations. This *singular* contribution does not disappear, even when higher order corrections vanish. This fact can be illustrated with a simple example. Let us consider the following Lagrangian:

$$\tilde{L} = L(\dot{q}) + \varepsilon^2 \, l(\ddot{q}), \tag{1}$$

where

$$\begin{aligned} L(\dot{q}) &= \frac{\dot{q}^2}{2}, \\ l(\ddot{q}) &= (-1)^k \, \frac{\ddot{q}^2}{2}, \end{aligned}$$

$k = 1, 2$. Corresponding Euler-Lagrange equations are:

$$\ddot{q} - (-1)^k \, \varepsilon^2 \, q^{(4)} = 0.$$

Let the initial conditions be

$$q(0) = q_o,$$
$$\dot{q}(0) = v_o,$$
$$\ddot{q}(0) = 0,$$
$$q^{(3)}(0) = \delta q.$$

As a result, one obtains two types of solutions. When $k = 1$ it has the form

$$q = q_o + v_o t - \delta q \, \varepsilon^3 \, \sin(t/\varepsilon)$$

and tends to the nonperturbed solution $q = q_o + v_o t$ when $\varepsilon \to 0$. When $k = 2$ it looks like

$$q = q_o + v_o t + \delta q \, \varepsilon^3 \, \mathrm{sh}(t/\varepsilon)$$

and *has a singular contribution even when* $\varepsilon \to 0$.

One meets theories with higher order curvature corrections rather often. One of the first examples is the wellknown Wheeler-Feynman electrodynamics [1]. It is based on the action that is parametrized in the form of two dimensional integral. Such integral can be approximated by the set of one dimensional integrals, so, an infinite set of time derivatives appears [2]. Modified theories become unlocal and lead to the equations with the higher order derivatives up to the infinite orders. It is important that there is no problem of singular contribution from higher derivatives because the solutions are limited and are infinitely differentiable. Therefore, the lagrangian of infinite order approximates the lagrangian of second order which provides the regular contribution of higher order corrections.

We would like to point out that the problems with new degrees of freedom, and, so, appearing of nonphysical "runaway" solutions are really actual for one special class of the models. There are the models where the corrections contain high but finite order of derivatives [3], where the singular contribution of higher order correction is not compensated by the contribution from next order(s). The most natural approach to such class of models is to introduce additional limits on solutions availability and to consider only such ones that have a regular part (expanding in Taylor series) relatively the numerical parameter before the higher derivatives [4]. If one considers such condition as necessary one there would be no problem of higher order corrections (In Ref. [4] the problem is treated in such a manner).

Here it is necessary to emphasis that higher order curvature corrections do not automatically cause singular unlimited solutions (see the example higher, Eq. (1)). So,

it is desirable to study the conditions of appearance of new "runaway solutions" and their status in the four dimensional (4d) gravity models with higher curvature corrections caused by string theory more carefully. It is important to extract the properties of the solutions (in this paper we restrict our consideration with spherically symmetric ones) when only limited (finite) number of corrections are considered. Further, during consideration of real physical problems it would be better to restrict consideration with models where higher order corrections (that are necessary to make a little step from General Relativity boundaries of applicability) do not cause singular contribution to the regular part of the solution. The stability of the solution under the initial conditions on the infinity is strongly required from the physical grounds.

In the perturbational approach, the 4d effective (string inspired) gravity action has the following form (we work in Planckian system of units where $\hbar = c = m_{Pl} = 1$) [5]:

$$\begin{aligned} S &= \frac{1}{16\pi} \int d^4x \sqrt{-g} \Big[-m_{Pl}^2 R + 2\partial_\mu \phi \partial^\mu \phi \\ &+ \lambda e^{-2\phi} l_2 + \lambda^2 e^{-4\phi} l_3 + \lambda^3 e^{-6\phi} l_4 + O(\lambda^4) \Big], \end{aligned} \tag{2}$$

where R is the Ricci scalar, ϕ is a dilatonic field, λ is a string coupling constant (an expansion parameter which is proportional to string tension α' with the numerical coefficient depending upon the type of string gravity) and l_i $(i = 2, 3, \ldots)$ are higher order curvature corrections. In this paper we study the class of the models where higher order curvature corrections consist from Riemannian tensor $R_{\mu\nu\alpha\beta}$ products. Such type of models are widely discussed now from different aspects [6]. This study focuses on asymptotically flat spherically symmetric static black hole solutions.

It should also be pointed out that the zeroth measure of the initial conditions manifold (only asymptotically flat, for example) is not very realistic, especially when new observational results in cosmology are taken into account. The highly probable existence of a global nonvanishing cosmological constant Λ requires the extension for the manifold of initial conditions. One can neglect the Λ influence on the global structure of the solution because it has zeroth order contribution relatively higher derivatives, so, it's influence is important only in the regions with rather small curvature. In black hole topologies where its influence is neglectible, it is possible to restrict the consideration by the requirement of stability (of Lyapunov type) under the initial conditions (and, hence, to take Λ existence into account) at the infinity.

The most convenient choice of four dimensional metric is, therefore,

$$ds^2 = -\Delta(r) dt^2 + \frac{\sigma^2(r)}{\Delta(r)} dr^2 + r^2 \Big(d\theta^2 + \sin^2\theta d\varphi^2 \Big). \tag{3}$$

The solution of corresponding nonperturbed field equations is the wellknown Schwarzschild one

$$\Delta = \Delta^0(r) = 1 - \frac{2M}{r},$$

$$\begin{aligned}\sigma &= \sigma^0(r) = 1, \\ \phi &= \phi^0(r) = \phi_0 = \text{const},\end{aligned} \tag{4}$$

where M is black hole mass.

When the correction $l_2 = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2$ in (2) is taken into account (which is the wellknown 4d curvature invariant named as Gauss-Bonnet term), the corresponding solutions do not contain higher derivatives. It is necessary to emphasize that the discussed black hole solution behavior strongly differs from usual Schwarzschild one under the event horizon as it was shown in detail in Ref. [7]. When the metric (3) is substituted in (2), the action becomes [8]

$$S = \int dr \Big[L_\Upsilon(r, \Delta, \sigma, \phi, \Delta', \phi', \lambda) + \lambda^2 l_\Upsilon(r, \Delta, \sigma, \phi, \Delta', \sigma', \Delta'', \lambda) \Big] \tag{5}$$

where $\Upsilon = b, h, s$ corresponds to the three string theory types considered in this paper: bosonic (b), heterotic (h) and superstring II (s).

The basic Lagrangian does not contain higher derivatives,

$$\begin{aligned} L_b &= L_h = L_0(r, \Delta, \sigma, \phi, \Delta', \phi') + \lambda L_{GB}(r, \Delta, \sigma, \phi, \Delta', \phi'), \\ L_s &= L_0 = -\frac{\Delta' r + \Delta + \sigma^2 - \Delta r^2 (\phi')^2}{\sigma}, \\ L_{GB} &= 4e^{-2\phi}\phi'\Delta' \frac{\Delta(\phi')^2 - \sigma^2}{\sigma^3}. \end{aligned} \tag{6}$$

L_0 is simply the contribution resulting from General Relativity.

The next order curvature corrections to l_Υ have the following form [9]:

$$\begin{aligned} l_b &= C_1 \sqrt{-g} e^{-4\phi} (2R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\gamma\rho} R^{\gamma\rho}{}_{\mu\nu} - 4R^{\mu\nu}{}_{\alpha\beta} R_\nu{}^{\gamma\beta\rho} R^\alpha{}_{\rho\mu\gamma} \\ &+ \frac{3}{2} R R^2_{\mu\nu\alpha\beta} + 12 R^{\mu\nu\alpha\beta} R_{\alpha\mu} R_{\beta\nu} + 8 R^{\mu\nu} R_{\nu\alpha} R^\alpha{}_\mu - 12 R R^2_{\alpha\beta} + \frac{1}{2} R^3 \\ &+ R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\gamma\rho} R^{\gamma\rho}{}_{\mu\nu}) + O(\lambda), \\ l_h &= C_2 \sqrt{-g} e^{-6\phi} (A - \frac{1}{8} (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta})^2 - \frac{1}{4} R_{\mu\nu}{}^{\gamma\delta} R_{\gamma\delta}{}^{\rho\eta} R_{\rho\eta}{}^{\alpha\beta} R_{\alpha\beta}{}^{\mu\nu} \\ &+ \frac{1}{2} R_{\mu\nu}{}^{\alpha\beta} R_{\alpha\beta}{}^{\rho\eta} R^\mu{}_{\eta\gamma\delta} R_\rho{}^{\nu\gamma\delta} + R_{\mu\nu}{}^{\alpha\beta} R_{\alpha\beta}{}^{\rho\nu} R_{\rho\eta}{}^{\gamma\delta} R_{\gamma\delta}{}^{\mu\eta}) + O(\lambda), \\ l_s &= C_3 \sqrt{-g} e^{-6\phi} A + O(\lambda), \end{aligned} \tag{7}$$

where

$$A = \zeta(3) [R_{\mu\nu\rho\eta} R^{\alpha\nu\rho\beta} (R^{\mu\nu}{}_{\delta\beta} R_{\alpha\gamma}{}^{\delta\eta} - 2R^{\mu\nu}{}_{\delta\alpha} R_{\beta\gamma}{}^{\delta\eta})],$$

$R^2_{\mu\nu} = R_{\mu\nu} R^{\mu\nu}$, $\zeta(3)$ is Riemanian zeta-function, $C_{1,2,3}$ are numerical coefficients.

The influence of higher order curvature corrections to the behavior of spherically symmetric static solutions was considered in [8] where a perturbed solution close the nonperturbed one was studied. It was obtained by a specially developed and coded numerical iteration method. The only indefinite point of the perturbed solution occurred

near the particular point r_s (internal black hole singularity of a new type at finite distance from the origin, provided by Gauss-Bonnet term, see Ref. [7]). If one works in the frames of such an approach, the singular contribution disappears. From a pure mathematical point of view, the solution tends to some particular branch that begins from a particular manifold of initial conditions [10] with null measure. Even small changes of initial conditions cause an appearance of a singular contribution that does not vanish when $\varepsilon \to 0$.

Finally, the main aim of this paper is to understand under which circumstances a singular contribution is induced in three different closed 4d low energy string gravity models on the Schwarzschild metric background when radial coordinate r is rather large, when the only *finite* number of higher order curvature corrections are taken into account (as the complete expansion is unknown). The paper is organized as follows: In Section 2 we show some general mathematical considerations of higher order singular corrections behavior, in Section 3 we apply these results to bosonic, heterotic and superstring II (SUSY II) closed 4d low energy models, Section 4 contains the SUSY II numerical investigations, Section 5 is discussions and conclusions one.

2. General theory of singular contributions

Models with corrections including higher order derivatives can be generically written as:

$$\tilde{L}(t, x, \dot{x}, \ddot{x}) = L(t, x, \dot{x}) + \varepsilon^2 l(t, x, \dot{x}, \ddot{x}), \tag{8}$$

where ε is an expansion parameter, t is real variable, $x(t)$ is a smooth vector of n-dimensional manifold and L, l are smooth functions.

The corresponding Euler-Lagrange equations are

$$\frac{\partial \tilde{L}}{\partial x} - \frac{d}{dt}\frac{\partial \tilde{L}}{\partial \dot{x}} + \frac{d^2}{dt^2}\frac{\partial \tilde{L}}{\partial \ddot{x}} = 0.$$

One can rewrite these equations as follows

$$\begin{aligned} &\varepsilon^2 x^{(4)} \frac{\partial^2 l}{\partial \ddot{x} \partial \ddot{x}} = \ddot{x} \frac{\partial^2 L}{\partial \dot{x} \partial \dot{x}} + \Phi(t, x, \dot{x}) + \\ &+ \varepsilon^2 \left[-x^{(3)} \frac{\partial^3 l}{\partial \ddot{x} \partial \ddot{x} \partial \ddot{x}} x^{(3)} + x^{(3)} \Xi(t, x, \dot{x}, \ddot{x}) + \Psi(t, x, \dot{x}, \ddot{x}) \right], \end{aligned} \tag{9}$$

where $\dfrac{\partial^2 L}{\partial \dot{x} \partial \dot{x}}$ and $\dfrac{\partial^2 l}{\partial \ddot{x} \partial \ddot{x}}$ are symmetric matrixes of second order derivatives of L and l (respectively) whereas Φ, Ξ and Ψ are vector matrixes with second and third derivatives of L and l (respectively).

Introducing the following change of variables: $\dot{x} = y$, $\ddot{x} = z$, $\varepsilon x^{(3)} = v$ and

considering the matrix $\frac{\partial^2 l}{\partial \ddot{x} \partial \ddot{x}}$ as a nondegenerated one, we can write (9) in the form

$$\begin{cases} \varepsilon \dot{v} = F(t,x,y,z,\varepsilon), \quad \varepsilon \dot{z} = v, \\ \dot{y} = z, \quad \dot{x} = y. \end{cases} \tag{10}$$

where

$$\begin{aligned} F(t,x,y,z,\varepsilon) &= \left[\frac{\partial^2 l}{\partial z \partial z}(t,x,y,z)\right]^{-1} z \frac{\partial^2 L}{\partial y \partial y}(t,x,y) + \left[\frac{\partial^2 l}{\partial z \partial z}\right]^{-1} \Phi(t,x,y) \\ &- \left[\frac{\partial^2 l}{\partial z \partial z}\right]^{-1} v \frac{\partial^3 l}{\partial z \partial z \partial z} v + \varepsilon \left[\frac{\partial^2 l}{\partial z \partial z}\right]^{-1} v \Xi(t,x,y,z) \\ &+ \varepsilon^2 \left[\frac{\partial^2 l}{\partial z \partial z}\right]^{-1} \Psi(t,x,y,z). \end{aligned}$$

When $\varepsilon = 0$ the system (10) has a degenerated solution $\overset{0}{X}$,

$$x = x^0(t), \; y = y^0(t), \; z = z^0(t), \; v = v^0(t) \equiv 0, \tag{11}$$

which corresponds to the nonperturbed one of Euler-Lagrange equations (8) when the initial conditions are $x^0(t_0) = x_0$, $y^0(t_0) = y_0$. This means that the equations (10) belong to A.N.Tikhonov's class [11]. According to the related theorem, the closeness of perturbed and nonperturbed solutions can be obtained, with initial conditions in the neighborhood of x_0, y_0, $x''(t_0)$, $x'''(t_0)$, if all the real parts of eigenvalues ω of matrix

$$\begin{pmatrix} 0 & \dfrac{\partial F}{\partial z} \\ & \\ I & 0 \end{pmatrix}$$

are negative. This matrix is calculated on nonperturbed solutions $\overset{0}{X}$ (11), I is identity $n \times n$ matrix.

After some algebra, the equation on eigenvalues ω can be represented as

$$\det \left| \omega^2 \frac{\partial^2 l}{\partial z \partial z} - \frac{\partial^2 L}{\partial y \partial y} \right|_{\overset{0}{X}} = 0.$$

Therefore, to provide the convergence of the perturbed solution to the nonperturbed one, it is possible to require only $Re(\omega) = 0$. This corresponds to the case when the value

$$u(t) \leq 0, \tag{12}$$

where $u(t)$ can be calculated from the following equation

$$\det \left| u(t) \frac{\partial^2 l}{\partial \ddot{x} \partial \ddot{x}} - \frac{\partial^2 L}{\partial \dot{x} \partial \dot{x}} \right|_{X = \overset{0}{X}(t)} = 0. \tag{13}$$

We shall call $u(t)$ the *singular index*. Eq. (13) is valid on the interval $t \in [t_0, T]$ where the solution of nonperturbed problem exists when $\varepsilon = 0$.

It is important to emphasize that the condition (12) is not enough to ensure the absence of any singular contribution to Eqs. (8), but when the condition (12) is broken a singular contribution of order $\exp\left(\frac{t}{\varepsilon}\,\mathrm{Re}\sqrt{u(t)}\right)$ appears necessarily.

Switching to the string gravity perturbed Lagrangian

$$\tilde{L} = L(r, \Delta, \sigma, \phi, \Delta', \phi') + \varepsilon^2\; l(r, \Delta, \sigma, \phi, \Delta', \sigma', \Delta'')$$

and taking into account that $\phi(r)$ is not perturbed, one can conclude that the condition (12) on $u(r)$ can be fulfilled if

$$|u\,P - Q|_{\Delta=\Delta^0(r),\ \sigma=\sigma^0(r),\ \phi=\phi^0(r)} = 0, \tag{14}$$

where

$$P = \begin{pmatrix} l_{\Delta''\Delta''} & l_{\sigma'\Delta''} \\ l_{\Delta''\sigma'} & l_{\sigma'\sigma'} \end{pmatrix}$$

and

$$Q = \begin{pmatrix} L_{\Delta'\Delta'} - L_{\Delta'\phi'}\left[L_{\phi'\phi'}\right]^{-1} L_{\phi'\Delta'} & L_{\sigma\Delta'} - L_{\sigma\phi'}\left[L_{\phi'\phi'}\right]^{-1} L_{\phi'\Delta'} \\ L_{\Delta'\sigma} - L_{\Delta'\phi'}\left[L_{\phi'\phi'}\right]^{-1} L_{\phi'\sigma} & L_{\sigma\sigma} - L_{\sigma\phi'}\left[L_{\phi'\phi'}\right]^{-1} L_{\phi'\sigma} \end{pmatrix}$$

3. Singular indexes of low energy effective string gravity

To investigate the problem of singular contributions in string theory Lagrangians with higher order curvature corrections in spherically symmetric space times ,it is necessary, as a first step, to apply the formulas (6) and (7) to determine the accurate form of the main Lagrangian and higher order curvature corrections. The software packages from MAPLE and REDUCE were specially developed and used to obtain the corresponding singular indexes. With the help of Eq. (14) and after the substitution of Schwarzschild values of metric functions (4) in the limit $r/M \to \infty$, we get the following asymptotic formulas.

For bosonic case

$$u^b_{1,2} = \frac{e^{4\phi_0} r^2 (1 \pm i\sqrt{15})}{576}\left[\frac{r}{M} + o\left(\frac{r}{M}\right)\right].$$

For heterotic case

$$u^h_1 = 0.008773 e^{6\phi_0} r^4 \left[\frac{r^2}{M^2} + o\left(\frac{r^2}{M^2}\right)\right],$$
$$u^h_2 = -0.02291 e^{6\phi_0} r^4 \left[\frac{r^2}{M^2} + o\left(\frac{r^2}{M^2}\right)\right]$$

For SUSY II case

$$u_1^s = -0.03496 e^{6\phi_0} r^4 \left[\frac{r^2}{M^2} + o\left(\frac{r^2}{M^2}\right)\right],$$
$$u_2^s = -0.198 e^{6\phi_0} r^4 \left[\frac{r^2}{M^2} + o\left(\frac{r^2}{M^2}\right)\right]$$

Considering these asymptotical formulae, one can conclude that only SUSY II does not induce any singular contribution to the solution in the neighborhood of the Schwarzschild one , even for big r. To understand the perturbed solution behavior in SUSY II case, it is necessary to perform a numerical investigation of this solution dependence upon different sets of initial conditions $\Delta''(r_0)$, $\Delta'''(r_0)$, $\sigma'(r_0)$, $\sigma''(r_0)$, that are close to the corresponding Schwarzschild values $\Delta_0''(r) = -4M/r^3$, $\Delta_0'''(r) = 12M/r^4$, $\sigma_0'(r) = \sigma_0'''(r) = 0$.

4. Results of numerical investigation of perturbed solutions in SUSY II

To investigate the difference between the perturbed SUSY II solution and the nonperturbed Schwarzschild one , we solved Eqs. (9) as the Cauchy problem starting from initial point r_0 using initial conditions from accurate Schwarzschild solution. A 7th order Runge-Kutta code was especially chosen for the integration of the resulting system. Some computing problems occurred in the range of big r_0/M. For instance, when $\lambda = 0.1$, $M = 1$ for $r_0 > 6$ (we work in Planckian units) the system becomes so rigid that it can not be solved. We could integrate these equations only in the range that was situated up to the event horizon at $r/M \in [2.1, 5.2]$ and for $M < 100$, because for higher mass values and higher r/M values local influence of the corrections became so small that calculations stopped at the first integration step.

For different values of r_0, the difference of $\Delta(r)$, $\sigma(r)$ and $\phi(r)$ from corresponding Schwarzschild values with high accuracy (not less than integration method one) were proportional to $(r - r_0)^2$, so,

$$\delta x = C(r - r_0)^2, \tag{15}$$

where C is numerical coefficient.

We investigated the dependence of C as a function of the initial integration point r_0 and mass M when λ is fixed and equal to 0.1†. So, the behavior of $C(r_0, M)$ is described by the following fit

$$C(r_0, M) = \alpha(M)\left(\frac{r_0}{M}\right)^{-\beta(M)}.$$

† The increasing of M when λ is fixed is equivalent (up to dimensionality) to decreasing of λ when M is fixed

It is important to emphasize that $\beta(M) > 10$ has very weak dependence upon M. For instance, the results for $\phi(r)$ ($\phi_0(r) \neq 0$) are presented in the Table 1.

M	α	β
1	134.	13.53
2	0.411	13.59
15	$4.39 \cdot 10^{-8}$	13.81
50	$3.29 \cdot 10^{-12}$	13.81
100	$1.49 \cdot 10^{-14}$	13.92

Table 1. The dependence of the numerical fit coefficients of ϕ expansion α and β versus mass M

If one extrapolates formula (15) for δx (difference between perturbed SUSY II solution and nonperturbed Schwarzschild one) in the complete range of $r/M \in (2, \infty)$, after obtaining the asymptotically flat solution (starting from $r_0 \to \infty$), one would conclude (for the fixed r and β from Table 1) that:

$$\lim_{r_0 \to \infty} C(r_0, M)(r - r_0)^2 = \lim_{r_0 \to \infty} C(r_0, M) r_0^2 \left(1 + o\left(\frac{r}{r_0}\right)\right) = 0.$$

Then it is possible to estimate $\delta x(r)$ as

$$\delta x(r) < \max_{r_0 \in [r, \infty)} C(r_0, M)(r - r_0)^2 = \frac{4\alpha(M)}{(\beta - 2)^2} \left(\frac{\beta}{\beta - 2}\frac{r}{M}\right)^{-\beta} r^2.$$

For example, at the event horizon $r = r_h = 2M$:

$$\delta x(2M) < D(\beta)\alpha(M)M^2,$$

where

$$D(\beta) = \frac{16}{(\beta - 2)^2} \left(\frac{2\beta}{\beta - 2}\right)^{-\beta}.$$

If the value of parameter β does not differ apparently from those presented in Table 1 and the dependence $\alpha(M)$ is approximately the same, one can conclude that

$$\lim_{M \to \infty} \alpha(M)M^2 = 0.$$

So, the value of the perturbation becomes zero when relative weight of higher order corrections vanishes.

Finally, this, as it seems to the authors of the paper, means that the 4th order curvature correction in SUSY II does not provide any appreciable differences from the Schwarzschild solution, at least up to the event horizon.

5. Discussion and conclusions

Based on our preliminary results in the framework of models with higher order curvature corrections made of pure Riemanian tensor products, it is reasonable to assume that the only model that does not provide any singular contribution is the SUSY II one. As curvature corrections of next orders do not produce new higher derivatives, in a case if singular contributions are absent in the 4th perturbation order, they would be absent in all perturbative orders. It is impossible to draw more accurate conclusions in this framework. Our results show the existence of singular contributions in bosonic and heterotic realizations of string gravity and give hope that SUSY II model can be free from singular contributions, at least for spherically symmetric static space times.

It is necessary to emphasis that the loop expansion of string gravity (the origin of higher order curvature corrections) initially consists from the terms with increasing order of derivatives [9, 12]. But working in the frames only of one considered approximation (for example, 3rd, 4th, ...) it is possible to make some manipulations [9, 12] taking usage from the fact that corrections contain the products with dilaton. So, analogously to the transformation that was used when the Gauss-Bonnet term by itself was changed to its current [7], one can make partial integration of the correction according to the scheme:

$$\int e^{n\Phi}\nabla_n R^n = (e^{n\Phi}\nabla_{n-1}R^n) - \int n\nabla\Phi e^{n\Phi}\nabla_{n-1}R^n. \tag{16}$$

Making this procedure "n" times one can shift differentiation from Riemannian tensor to the dilaton and finally result with the terms like R^n_{ijkl} without derivatives. Then, as one is working in the limits of corresponding correction, the term with dilaton derivatives can be transferred to a next order as it is shown in Formula (10) at Ref. [13]. Hence, *in the frames of chosen order the expression becomes free from derivatives.*

So, the result of our study is that when the limited number of higher order curvature corrections are taken into account a spherically symmetric solution can disappear. Finally, the singular contribution of the higher order curvature corrections in the low energy limit of string gravity, in the frames of Lagrange approach, represents a problem. Anyway, as the consideration of higher order corrections is necessary to extend the boundaries of applicability of classical gravity (and to obtain preliminary directions on some effects, that have quantum gravity nature) this can be avoided by two possible ways:

- to suggest additional principles for choosing the special initial conditions for the higher derivatives, providing the closeness of the perturbed solution to the nonperturbed one (at least for big r) that include all the physically interesting cases (asymptotically flat, dS/AdS);
- not to restrict the consideration by the frames of chosen perturbative order and consider the "rest term" of series expansion (this means that the application of

Formula (10) from [13] and all the same ones are not directly allowed in black hole cases).

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